Grade Eight

As students enter eighth grade they have written and interpreted expressions, solved equations and inequalities, explored quantitative relationships between dependent and independent variables, and solved problems involving area, surface area, and volume. Students have also begun to develop an understanding of statistical thinking (Adapted from The Charles A. Dana Center Mathematics Common Core Toolbox 2012).

WHAT STUDENTS LEARN IN GRADE EIGHT

[Note: Sidebar]

Grade Eight Critical Areas of Instruction

In grade eight, instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence and understanding and applying the Pythagorean Theorem.

Students also work towards fluency with solving simple sets of two equations with two unknowns by inspection.

Grade Eight Standards for Mathematical Content

The Standards for Mathematical Content emphasize key content, skills, and practices at each grade level and support three major principles:

- **Focus:** Instruction is focused on grade level standards.
- **Coherence:** Instruction should be attentive to learning across grades and should link major topics within grades.
- **Rigor:** Instruction should develop conceptual understanding, procedural skill and fluency, and application.

Grade level examples of focus, coherence, and rigor will be indicated throughout the chapter.

Not all of the content in a given grade is emphasized equally in the standards. Cluster headings can be viewed as the most effective way to communicate the **focus and coherence** of the standards. Some clusters of standards require a greater instructional emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the later demands of college and career readiness.

The following Grade 8 Cluster-Level Emphases chart highlights the content emphases in the standards at the cluster level for this grade. The bulk of instructional time should be given to “Major” clusters and the standards within them. However, standards in the “Supporting/Additional” clusters should not be neglected. To do so will result in gaps in students’ learning, including skills and understandings they may need in later grades. Students need opportunities to reinforce topics in major clusters at a grade by utilizing topics in the supporting and additional clusters. Instruction should include problems and activities that support natural connections between the clusters.

Teachers and administrators alike should note that the standards are not topics to be checked off a list during isolated units of instruction, but rather content to be developed throughout the school year through rich instructional experiences and presented in a coherent manner (Adapted from the Partnership for Assessment of Readiness for College and Careers [PARCC] 2012).

[**Note**: The Emphases chart should be a graphic inserted in the grade level section. The explanation “key” needs to accompany it.]

Grade 8 Cluster-Level Emphases

The Number System

- [a/s]: Know that there are numbers that are not rational, and approximate them by rational numbers[1]. **(8.NS.1-2)**

[1] Work with the number system in this grade is intimately related to work with radicals, and both of these may be connected to the Pythagorean Theorem as well as to volume problems, e.g., in which a cube has known volume but unknown edge lengths.

Expressions and Equations

- [m]: Work with radicals and integer exponents. **(8.EE.1-4▲)**
- [m]: Understand the connection between proportional relationships, lines, and linear equations. **(8.EE.5-6▲)**
- [m]: Analyze and solve linear equations and pairs of simultaneous linear equations. **(8.EE.7-8▲)**

Functions

- [m]: Define, evaluate, and compare functions. **(8.F.1-3▲)**
- [a/s]: Use functions to model relationships between quantities.[2] **(8.F.4-5)**

Geometry

- [m]: Understand congruence and similarity using physical models, transparencies, or geometry software. **(8.G.1-5▲)**
- [m]: Understand and apply the Pythagorean Theorem. **(8.G.6-8▲)**
- [a/s]: Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres. **(8.G.9)**

Statistics and Probability

- [a/s]: Investigate patterns of association in bivariate data.[3] **(8.SP.1-4)**

Explanations of Major, Additional and Supporting Cluster-Level Emphases
Major[4] **[m]** (▲) clusters – areas of intensive focus where students need fluent understanding and application of the core concepts. These clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness.
Additional [a] clusters – expose students to other subjects; may not connect tightly or explicitly to the major work of the grade **Supporting [s]** clusters – rethinking and linking; areas where some material is being covered, but in a way that applies core understanding; designed to support and strengthen areas of major emphasis. *A Note of Caution: Neglecting material will leave gaps in students' skills and understanding and will leave students unprepared for the challenges of a later grade.

(Adapted from Smarter Balanced Assessment Consortia [Smarter Balanced], DRAFT Content Specifications 2012).

[2] The work in this cluster involves functions for modeling linear relationships and a rate of change/initial value, which supports work with proportional relationships and setting up linear equations.

[3] Looking for patterns in scatterplots and using linear models to describe data is directly connected to the work in the Expressions and Equations clusters. Together, these represent a connection to the Standard for Mathematical Practice 4, Model with mathematics.

[4] The ▲symbol will indicate standards in a Major Cluster in the narrative.

Connecting Mathematical Practices and Content

The Standards for Mathematical Practice (MP) are developed throughout each grade and, together with the content standards, prescribe that students experience mathematics as a rigorous, coherent, useful, and logical subject that makes use of their ability to make sense of mathematics. The MP standards represent a picture of what it looks like for students to understand and do mathematics in the classroom and should be integrated into every mathematics lesson for all students.

Although the description of the MP standards remains the same at all grades, the way these standards look as students engage with and master new and more advanced mathematical ideas does change. Below are some examples of how the MP standards may be integrated into tasks appropriate for grade eight students. (Refer to pages 9–12 in the Overview of the Standards Chapters for a complete description of the MP standards.)

Standards for Mathematical Practice (MP)
Explanations and Examples for Grade Eight

Standards for Mathematical Practice	Explanation and Examples
MP.1 Make sense of problems and persevere in solving them.	In grade eight, students solve real-world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, “What is the most efficient way to solve the problem?”, “Does this make sense?”, and “Can I solve the problem in a different way?”
MP.2 Reason abstractly and quantitatively.	Students represent a wide variety of real-world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. They examine patterns in data and assess the degree of linearity of functions. Students *contextualize* to understand the meaning of the number or variable as related to the problem and *decontextualize* to manipulate symbolic representations by applying properties of operations.
MP. 3 Construct viable arguments and critique the reasoning of others.	Students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (e.g., box plots, dot plots, histograms). They further refine their mathematical

	communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like "How did you get that?", "Why is that true?" and "Does that always work?" They explain their thinking to others and respond to others' thinking.
MP.4 Model with mathematics.	Eighth grade students model real-world problem situations symbolically, graphically, in tables, and contextually. Working with the new concept of a *function*, students learn that relationships between variable quantities in the real world often satisfy a dependent relationship, in that one quantity determines the value of another. Students form expressions, equations, or inequalities from real-world contexts and connect symbolic and graphical representations. Students use scatterplots to represent data and describe associations between variables. They should be able to use any of these representations as appropriate to a problem context. Students should be encouraged to answer questions, such as "What are some ways to represent the quantities?" or "How might it help to create a table, chart, graph,…?"
MP.5 Use appropriate tools strategically.	Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade eight may translate a set of data given in tabular form to a graphical representation to compare it to another data set. Students might draw pictures, use applets, or write equations to show the relationships between the angles created by a transversal that intersects parallel lines. Teachers might ask, "What approach are you considering trying first?" or "Why was it helpful to use…?"
MP.6 Attend to precision.	In grade eight, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to the number system, functions, geometric figures, and data displays. Teachers might ask "What mathematical language, definitions, properties…can you use to explain…?
MP.7 Look for and make use of structure.	Students routinely seek patterns or structures to model and solve problems. In grade eight, students apply properties to generate equivalent expressions and solve equations. Students examine patterns in tables and graphs to generate equations and describe relationships. Additionally, students experimentally verify the effects of transformations and describe them in terms of congruence and similarity.
MP.8 Look for and express regularity in repeated reasoning.	In grade eight, students use repeated reasoning to understand the slope formula and to make sense of rational and irrational numbers. Through multiple opportunities to model linear relationships, they notice that the slope of the graph of the linear relationship and the rate of change of the associated function are the same. As students repeatedly check whether points are on the line of slope 3 through the point (1, 2), they might abstract the equation of the line in the form $(y-2)/(x-1)=3$. Students divide to find

	decimal equivalents of rational numbers (e.g. $2/_3 = 0.\overline{6}$) and generalize their observations. They use iterative processes to determine more precise rational approximations for irrational numbers. Students should be encouraged to answer questions, such as “How would we prove that…?” or “How is this situation like and different from other situations using this operations?”

(Adapted from Arizona Department of Education [Arizona] 2012 and North Carolina Department of Public Instruction [N. Carolina] 2013)

Standards-based Learning at Grade Eight

The following narrative is organized by the domains in the Standards for Mathematical Content and highlights some necessary foundational skills from previous grades and provides exemplars to explain the content standards, highlight connections to the various Standards for Mathematical Practice (**MP**), and demonstrate the importance of developing conceptual understanding, procedural skill and fluency, and application. A triangle symbol (**▲)** indicates standards in the major clusters (refer to the Grade 8 Cluster-Level Emphases chart on page 2).

Domain: The Number System

In seventh grade, adding, subtracting, multiplying, and dividing rational numbers was the culmination of numerical work with the four basic operations. The number system continues to develop in grade eight, expanding to the real numbers by introducing irrational numbers, and develops further in high school, expanding to become the complex numbers with the introduction of imaginary numbers (Adapted from PARCC 2012).

The Number System 8.NS

Know that there are numbers that are not rational, and approximate them by rational numbers.

1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.
2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g. π^2). *For example, by truncating the decimal expansion of √2, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

In eighth grade, students learn that not all numbers can be expressed in the form a/b where a and b are positive or negative whole numbers with $b \neq 0$. Such numbers are called *irrational*, and students explore cases of both rational and irrational numbers and their decimal expansions to begin to understand the distinction. That rational numbers have eventually repeating decimal expansions is a direct result of how one uses long division to produce a decimal expansion.

Why rational numbers have terminating or repeating decimal expansions.

In each step of the standard algorithm to divide a by b, one determines a partial quotient and a remainder; the requirement being that each remainder is smaller than the divisor (b). In simpler examples students will have noticed or been led to notice that once a remainder is repeated, from that point onward the decimal repeats, as with $\frac{1}{6} = 0.16\mathbf{666}\ldots = 0.1\overline{6}$ or $\frac{3}{11} = 0.27\mathbf{2727}\ldots = 0.\overline{27}$. If one were to imagine using long division to convert the fraction $\frac{3}{13}$ to a decimal without going through the tedium of actually producing the decimal, one can reason that the possible remainders are 1 through 12. Consequently, by the thirteenth remainder we MUST obtain a remainder that has occurred already and therefore a decimal that is repeating.

The full reasoning for why the converse is true, that eventually repeating decimals represent numbers that are rational, is beyond the scope of this grade. But students can use algebraic reasoning to show that eventually repeating decimals represent rational numbers in certain simple cases. **(8.NS.1)**

Example: Convert the repeating decimal $0.\overline{18}$ into a fraction of the form a/b.

Solution: One method for converting such a decimal into a fraction is to set $N = 0.\overline{18} = 0.18181818\ldots$. If this is the case, then $100N = 18.\overline{18}$. Subtracting $100N$ and N, we obtain $99N$. But this means that $99N = 18.\overline{18} - .\overline{18} = 18$. Solving for N, we see that $N = 18/99 = 2/11$.

Since every decimal is of one of the two forms eventually repeating or non-repeating, this leaves irrational numbers as precisely those numbers whose decimal expansions do not have a repeating pattern. Students understand this informally in grade eight, and they experience approximating irrational numbers by rational numbers in simple cases.

Example: Finding better and better approximations of $\sqrt{2}$.

The following reasoning can be used to approximate simple irrational square roots.

- Since $1^2 < 2 < 2^2$, this means that $\sqrt{2}$ must be between 1 and 2.
- By guessing and checking, since $1.4^2 = 1.96$, and $1.5^2 = 2.25$, we know that $\sqrt{2}$ is between 1.4 and 1.5.
- By some more guessing and checking, and using a calculator, we see that since $1.41^2 = 1.9881$ and $1.42^2 = 2.0164$, we know that $\sqrt{2}$ is between 1.41 and 1.42.

Continuing in this manner yields better and better approximations of $\sqrt{2}$. Allowing students to investigate this process with their calculators, they can get some experience with the idea that the decimal expansion of $\sqrt{2}$ never repeats. In addition, students should graph their successive approximations on number lines to reinforce their understanding of the number line as a tool for representing the real numbers.

Ultimately, students should come to an informal understanding that the set of real numbers is comprised of rational numbers and irrational numbers. They will continue to work with irrational numbers and their rational approximations when solving equations such as $x^2 = 18$ and in problems involving the Pythagorean Theorem. In the Expressions and Equations domain that follows, students learn to use radicals to represent such numbers. (Adapted from California Department of Education *A Look at Grades Seven and Eight in California Public Schools: Transitioning to the Common Core State Standards in English Language Arts and Mathematics* [CDE Transition Document] 2012, Arizona 2012, and N. Carolina 2013).

[Note: Sidebar]

Focus, Coherence, and Rigor:

The standards in the grade eight domain “The Number System” support major work at the grade with the Pythagorean Theorem (**8.G.6-8▲**) and connection to volume problems **(8.G.9)**, e.g., in which a cube has known volume but unknown edge lengths.

Domain: Expressions and Equations

In grade seven students formulated expressions and equations in one variable, and they used these equations to solve problems and fluently solved equations of the form

$px + q = r$, $p(x + q) = r$. In grade eight, students apply their previous understandings of ratio and proportional reasoning to the study of linear equations and pairs of simultaneous linear equations, which is a critical area of instruction at the grade.

Expressions and Equations	8.EE

Work with radicals and integer exponents.

1. Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.
2. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.
3. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as* 3×10^8 *and the population of the world as* 7×10^9*, and determine that the world population is more than 20 times larger.*
4. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Grade eight students add the following properties of integer exponents to their repertoire of rules for transforming expressions, and they use these properties to generate equivalent expressions.

Properties of Integer Exponents
For any nonzero numbers a and b and integers n and m.
1. $a^n a^m = a^{n+m}$
2. $(a^n)^m = a^{nm}$
3. $a^n b^n = (ab)^n$
4. $a^0 = 1$
5. $a^{-n} = 1/a^n$

Since students have been focusing on place value relationships in the base-ten number system since elementary school, working with powers of 10 is a natural place to start investigating the patterns that give rise to these properties. However, general bases should be explored, and will foreshadow the study of exponential functions in higher mathematics courses.

Example: Students can reason about patterns to explore the properties of exponents.

Students fill in the blanks in the table shown and discuss with a neighbor any patterns they find:

	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}
Expanded	$2 \times 2 \times 2$	2×2	2	?	?	?	?
Evaluate	8	4	2	?	?	?	?

Students can reason about why the value of 2^0 should be 1, based on patterns they may see, such as that in the bottom row of the table, each value is 1/2 of the value to the left of it. Students should explore similar examples with other bases to arrive at the general understanding that $a^n = a \times a \times \cdots \times a$ (n factors), $a^0 = 1$, and $a^{-n} = 1/a^n$.

Generally, Mathematical Practice standard 3 (**MP.3**) calls for students to construct mathematical arguments; thus reasoning should be emphasized when it comes to learning the properties of exponents. For example, students can reason that $5^3 \times 5^2 = (5 \times 5 \times 5) \times (5 \times 5) = 5^5$. Students generalize the properties of exponents after many experiences working with them before they use them fluently.

Students do not learn the properties of rational exponents until high school. However, in grade eight they start to work systematically with the square root and cube root symbols, writing, for example, $\sqrt{64} = 8$ and $\sqrt[3]{5^3} = 5$. Since $\sqrt{p}$ is defined to only mean the positive solution to the equation $x^2 = p$ (when it exists), it is not correct to say that $\sqrt{64} = \pm 8$. However, a correct solution to $x^2 = 64$ would be $x = \pm\sqrt{64} = \pm 8$. Students in grade eight are not in a position to prove that these are the only solutions, but rather use informal methods such as guess and check to verify them (Progressions 6-8 EE 2011).

Students recognize perfect squares and cubes, understanding that non-perfect squares and non-perfect cubes are irrational. Students should generalize from many experiences that: (**MP.2, MP.5, MP.6, and MP.7**)

- Squaring a square root of a number returns the number back (e.g., $\left(\sqrt{5}\right)^2 = 5$)

- Taking the square root of the square of a number *sometimes* returns the number back (e.g., $\sqrt{7^2} = \sqrt{49} = 7$, while $\sqrt{(-3)^2} = \sqrt{9} = 3 \neq -3$)
- Cubing a number and taking the cube root can be considered inverse operations.

Students expand their exponent work as they perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Students use scientific notation to express very large or very small numbers. Students compare and interpret scientific notation quantities in the context of the situation, recognizing that the powers of ten indicated in quantities expressed in scientific notation follow the rules of exponents shown above. (Adapted from CDE Transition Document 2012, Arizona 2012, and N. Carolina 2013)

Example: Ants and Elephants. An ant has a mass of approximately 4×10^{-3} grams and an elephant has a mass of approximately 8 metric tons. How many ants does it take to have the same mass as an elephant?
(Note: 1 kg = 1000 grams, 1 metric ton = 1000 kg.)

Solution: To compare the masses of an ant and an elephant, we convert the mass of an elephant into grams:

$$8 \text{ metric tons } \times \frac{1000 \text{ kg}}{1 \text{ metric ton}} \times \frac{1000 \text{ g}}{1 \text{ kg}} = 8 \times 10^3 \times 10^3 \text{grams} = 8 \times 10^6 \text{grams}.$$

If we let N represent the number of ants that have the same mass as an elephant, then $(4 \times 10^{-3})N$ is their total mass in grams. This should equal 8×10^6 grams. This gives us a simple equation:

$$(4 \times 10^{-3})N = 8 \times 10^6 \text{ which means that } N = \frac{8 \times 10^6}{4 \times 10^{-3}} = 2 \times 10^{6-(-3)} = 2 \times 10^9$$

Thus, 2×10^9 ants would have the same mass as an elephant.
(Adapted from Illustrative Mathematics, 8.EE Ant and Elephant.)

[Note: Sidebar]

Focus, Coherence, and Rigor:
As students work with scientific notation, they learn to choose units of appropriate size for measurement of very large or very small quantities. (**MP.2, MP.5, MP.6**)

Expressions and Equations	8.EE
Understand the connections between proportional relationships, lines, and linear equations.	
5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two	

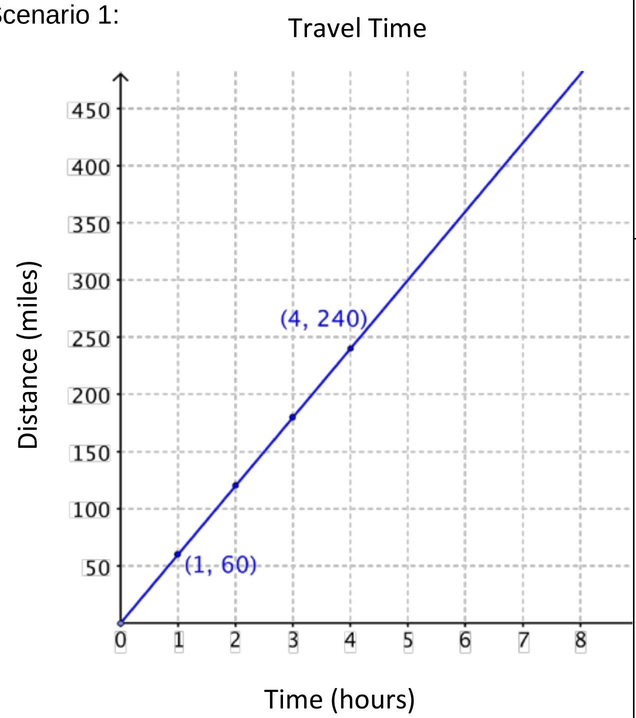
different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation and determine which of the two moving objects has greater speed.*

6. Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation *y* = *mx* for a line through the origin and the equation *y* = *mx* + *b* for a line intercepting the vertical axis at *b*.

Students build on their work with unit rates from sixth grade and proportional relationships in seventh grade to compare graphs, tables, and equations of proportional relationships (**8.EE.5▲**). Students identify the unit rate (or slope) to compare two proportional relationships represented in different ways (e.g., as graph of the line through the origin, a table exhibiting a constant rate of change, or an equation of the form $y = kx$). Students interpret the unit rate in a proportional relationship (e.g., r miles per hour) as the slope of the graph. They understand that the slope of a line represents a constant rate of change.

Example: Compare the scenarios below to determine which represents a greater speed. Include a description of each scenario that discusses unit rates in your explanation.

Scenario 1:



Scenario 2: The equation for the distance y in miles as a function of the time x in hours is:

$$y = 55x.$$

Solution: "The unit rate in Scenario 1 can be read from the graph; it is 60 miles per hour. In Scenario 2, I can see that this looks like an equation $y = kx$, and in that type of equation the unit rate is the constant k. Therefore the speed in Scenario 2 is 55 miles per hour. So the person traveling in Scenario 1 is going faster."

(Adapted from CDE Transition Document 2012, Arizona 2012, and N. Carolina 2013)

Following is an example of connecting the Standards for Mathematical Content with the Standards for Mathematical Practice.

Connecting to the Standards for Mathematical Practice—Grade Eight

Standard(s) Addressed

8.EE.5: Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in two different ways.

Example(s) and Explanations

Task: Below is a table that shows the costs of various amounts of almonds.

Almonds (pounds)	3	5	8	10	15
Cost (dollars)	15.00	25.00	40.00	50.00	75.00

1. Graph the cost versus the number of pounds of almonds. The number of pounds of almonds should be on the horizontal axis and the cost of the almonds on the vertical axis.
2. Use the graph to find the cost of 1 pound of almonds. Explain how you got your answer.
3. The table shows that 5 pounds of almonds costs \$25.00. Use your graph to find out how much 6 pounds of almonds costs.
4. Suppose that walnuts cost \$3.50 per pound. Draw a line on your graph that might represent the cost of different numbers of pounds of walnuts.
5. Which is cheaper? Almonds or walnuts? How do you know?

Solution:

1. A graph is shown.
2. To find the cost of 1 pound of almonds, one would locate the point that has first coordinate 1; this is the point (1, 5). This shows that the *unit cost* is \$5 per pound.
3. Students can do this by simply locating 6 pounds on the horizontal axis and finding the point on the graph associated with this number of pounds. However, the teacher can also urge students to notice that one can move along the graph by moving to the right 1 unit and noticing that we move 5 units up to the next point on the graph. This idea is the genesis of *slope* of a line and should be explored.
4. Ideally, students draw a line that passes through (0,0) and the approximate point (1, 3.50). Proportional thinkers might notice that 2 pounds of walnuts cost \$7, so they can plot a point with whole number coordinates.
5. Walnuts are cheaper. Students can explore several different ways to see this, including the unit cost, the steepness of the line, by comparing common quantities of nuts, etc.

Classroom Connections

The concept of slope can be approached in its simplest form with directly proportional quantities. In this case, when two quantities x and y are directly proportional, they are related by an equation $y = kx$, equivalently, $\frac{y}{x} = k$, where k is a constant known as the constant of proportionality. In the case of almonds above, the k in an equation would represent the unit cost of almonds. Students should have several experiences with graphing and exploring directly proportional relationships to build a foundation for understanding more general linear equations of the form $y = mx + b$.

Connecting to the Standards for Mathematical Practice

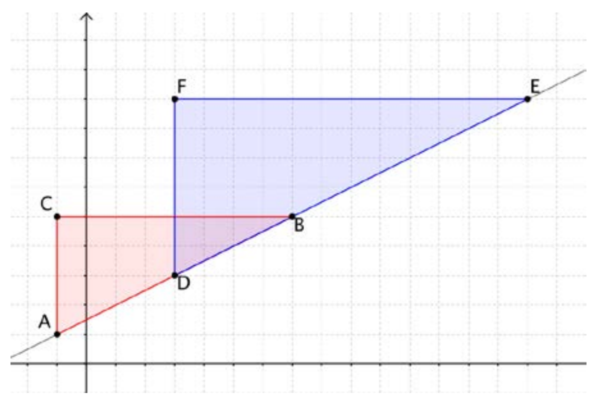
(MP.1) Students are encouraged to attack the entire problem and make sense in each step required.
(MP.4) Students are modeling a very simple real-life cost situation.

[Note: Sidebar]

Focus, Coherence, and Rigor:

The connection between the unit rate in a proportional relationship and the slope of its graph depends on a connection with the geometry of similar triangles. (See Standards **8.G.4-5▲**.) The fact that a line has a well-defined slope—that the ratio between the rise and run for any two points on the line is always the same—depends on similar triangles (Adapted from Progressions 6-8 EE 2011).

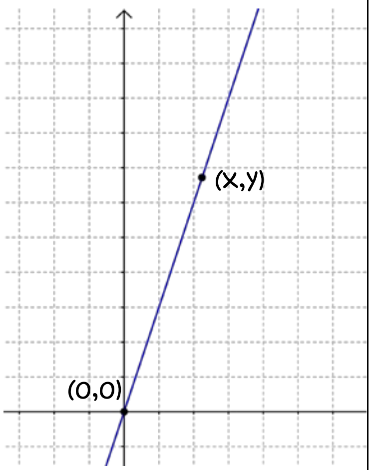
Standard **(8.EE.6▲)** represents a convergence of several ideas in this and previous grade levels. Students have graphed proportional relationships and found the slope of the resulting line, interpreting it as the unit rate **(8.EE.5▲)**. It is here that the language of “rise over run” comes into use. In the Functions domain, students will see that any linear equation $y = mx + b$ determines a function whose graph is a straight line (a *linear function*), and they verify that the slope of the line is equal to m (**8.F.3**). In standard **(8.EE.6▲)**, students go further and explain why the slope m is the same through any two points on a line. They justify this fact using similar triangles, which are studied in standards **(8.G.4-5▲)**.

Example of Reasoning (8.EE.6 ▲). Showing that the slope is the same between two points on a line.	
In seventh grade, students made scale drawings of figures and observed the proportional relationships between side lengths of such figures (**7.G.1▲**). In grade eight, students generalize this idea and study *dilations* of plane figures, and they define figures as being *similar* in terms of dilations (see standard **8.G.4▲**). It is discovered that similar figures share a proportional relationship between side lengths just like scale drawings did: there is a *scale factor* $k > 0$ such that corresponding side lengths of similar figures are related by the equation $s_1 = k \cdot s_2$. Furthermore, the ratio of two sides in one shape is equal to the ratio of the corresponding two sides in the other shape. Finally, in standard (**8.G.5▲**), students informally argue that triangles that have two corresponding angles of the same measure must be similar, and this is the final piece of the puzzle for the first result in standard (**8.EE.6▲**).	**Example:** "Explain why the slope between points A and B and points D and E are the same." **Solution:** "Angles $\angle A$ and $\angle D$ are equal since they are corresponding angles formed by the transversal crossing the vertical lines through points A and D. Since $\angle C$ and $\angle F$ are both right angles, the triangles are similar. This means the ratios $\frac{AC}{BC}$ and $\frac{DF}{EF}$ are equal. But when you find the 'rise over the run,' these are exactly the ratios that you find, and so the slope is the same between these two sets of points." 

In grade eight students build on previous work with proportional relationships, unit rates, and graphing to connect these ideas and understand that the points (x, y) on a non-vertical line are the solutions of the equation $y = mx + b$, where m is the slope of the line, as well as the unit rate of a proportional relationship in the case $b = 0$.

Example of Reasoning (8.EE.6). Deriving the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Example: “Explain how to derive the equation $y = 3x$ for the line of slope $m = 3$ shown below.” **Solution: “**I know that the slope is the same between any two points on a line. So I’ll choose the origin $(0,0)$ and a generic point on the line, calling it (x, y). By choosing a generic point like this I know that any point on the line will fit the equation I come up with. The slope between these two points is found by



$$3 = \frac{\text{rise}}{\text{run}} = \frac{y - 0}{x - 0} = \frac{y}{x}$$

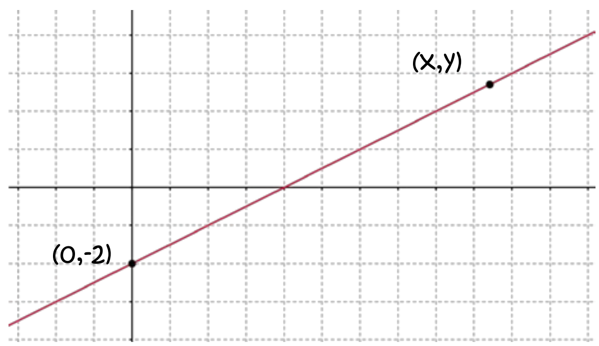
This equation can be rearranged to $y = 3x$.”

Example: “Explain how to derive the equation $y = \frac{1}{2}x - 2$ for the line of slope $m = \frac{1}{2}$ with intercept $b = -2$ shown below.”

Solution: “I know the slope is $\frac{1}{2}$ so I’ll calculate the slope using the point $(0, -2)$ and the generic point (x, y). The slope between these two points is found by

$$\frac{1}{2} = \frac{\text{rise}}{\text{run}} = \frac{y - (-2)}{x - 0} = \frac{y + 2}{x}$$

This can be rearranged to $y + 2 = \frac{1}{2}x$, which is the same as $y = \frac{1}{2}x - 2$.”



(Adapted from CDE Transition Document 2012, Arizona 2012, and N. Carolina 2013)

Expressions and Equations	8.EE

Analyze and solve linear equations and pairs of simultaneous linear equations.

7. Solve linear equations in one variable.
    a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equation of the form *x* = *a*, *a* = *a*, or *a* = *b* results (where *a* and *b* are different numbers).
    b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
8. Analyze and solve pairs of simultaneous linear equations.
    a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6.*

c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

Students have worked informally with one-variable linear equations as early as kindergarten. This important line of development culminates in grade eight as much of students' work involves analyzing and solving linear equations and pairs of simultaneous linear equations.

Grade eight students solve linear equations in one variable, including cases with one solution, infinitely many solutions, and no solutions (**8.EE.7▲**). Students show examples of each of these cases by successively transforming an equation into simpler forms ($x = a$, $a = a$, and $a = b$, where a and b represent different numbers). Solving some linear equations will require students to expand expressions using the distributive property and to collect like terms.

Solutions to One Variable Equations
• When an equation has only one solution, there is only one value of the variable that makes the equation true, e.g., with $12 - 4y = 16$. • When an equation has infinitely many solutions, the equation is true for all real numbers, and is sometimes referred to as an *identity*, e.g., with $7x + 14 = 7(x + 2)$. Solving this equation using familiar steps might yield $14 = 14$, a statement that is true no matter the value of x. Students should be encouraged to think about why this means that any real number solves the equation, and relate it to back to the original equation (e.g., the equation is just showing the distributive property). • When an equation has no solutions, we sometimes say the equation is *inconsistent*, e.g. with $5x - 2 = 5(x + 1)$. Attempting to solve this equation might yield $-2 = 5$, a false statement no matter the value of x. Students should be encouraged to reason why there are no solutions to the equation, for example, by observing that the original equation is equivalent to $5x - 2 = 5x + 5$, and reasoning that it is never the case that $N - 2 = N + 5$ no matter what N is.

Grade eight students also analyze and solve pairs of simultaneous linear equations. Solving pairs of simultaneous linear equations builds on the skills and understandings students used to solve linear equations with one variable (**8.EE.8 a-c▲**), and systems of linear equations can also have one solution, infinitely many solutions, or no solutions. Students will discover these cases as they graph systems of linear equations and solve them algebraically. Grade eight students learn that for a system of linear equations:

- If the graphs of the lines meet at one point (the lines intersect), then there is one solution, the ordered pair of the point of intersection representing the solution of the system.
- If the graphs of the lines do not meet (the lines are parallel), the system has no solutions, and the slopes of these lines are the same.
- If the graphs of the lines are *coincident* (the graphs are exactly the same line) then the system has infinitely many solutions, the solutions being the set of all ordered pairs on the line.

Example: Introducing Systems of Linear Equations.
To introduce the concept of a system of linear equations, the teacher might ask students to get into small groups and think about how they would start a business selling coffee at school during lunch. Then, each group would create a budget that details the cost of the items they would have to purchase each month (students could use the internet to acquire pricing or use their best estimate), as well as a monthly total. Each group would also establish a price for a cup of their coffee. Students can also discuss a model (equation) for the profit their business will make in a month. The teacher might ask students thoughtful questions, such as: 1. What are some variable quantities in our situation? (Important are cups of coffee sold, monthly profit.) 2. What is the profit at the beginning of the month? (A negative number corresponding to the monthly total of items purchased) 3. How many cups of coffee will you need to sell to make a profit? Instruct students to make a table that shows profit vs. cups of coffee sold, for multiples of 10 cups of coffee up to 200. Instruct students to create a graph from the data in their table. The teacher can demonstrate the graphs of the lines $y = 0$ and $y = 50$, then ask students to draw the same lines on their graph. Ask students the meaning of those lines. (Solution: the point when the business is no longer losing money, the point when the business is making money). The teacher

can demonstrate the points of intersection. Discuss the importance of those two coordinates. Finally, ask two students from different groups (pre-select groups whose graphs will intersect) to graph their data on the same axis for the whole class to see. Discuss the significance of the point of intersection of the two lines, including the concept that the number of cups sold and the profit will be the same at that point. As a class, students write equations for both lines and demonstrate by substitution that the coordinates of the intersection point are solutions to both equations.

By making connections between algebraic and graphical solutions and the context of the system of linear equations, students are able to make sense of their solutions.

Students solve real-world and mathematical problems leading to two linear equations in two variables. Below is an example of how reasoning about real-world situations can be used to introduce and make sense out of solving systems of equations by elimination. The technique of elimination can be used in general cases to solve systems of equations.

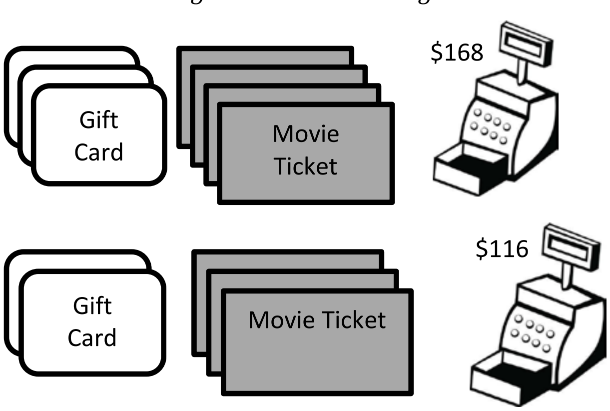
Example: Solving a System of Equations by Elimination.

Suppose you knew that 3 gift cards and 4 movie tickets altogether cost $188, while 2 gift cards and 3 movie tickets altogether cost $116.

1. Explain how to use this information to find the cost of one gift card and one movie ticket.
2. Next, explain how you could find the cost of 1 movie ticket.
3. Explain how you would find the cost of 1 gift card.

Solution:

1. If I let g represent the cost of a gift card and t represent the cost of a movie ticket, then I know that $3g + 4t = 168$ and $2g + 3t = 116$. I can represent this in a diagram:



I can see that if I subtract the 2 gift cards and 3 movie tickets from the 3 gift cards and 4 movie tickets, I get $168 – $116 = $52. This means the cost of 1 of each item together is $52. I can represent this by

$$\begin{array}{c} 3g + 4t = 168 \\ \underline{2g + 3t = 116} \\ g + t = 52 \end{array}$$

2. Now I can see that 2 of each item would cost $104. If I subtract this result from the second equation above, I am left with 1 movie ticket and it costs $12.

$$\begin{array}{c} 2g + 3t = 116 \\ \underline{2g + 2t = 104} \\ 1t = 12 \end{array}$$

3. Now it is easy to see that if 1 gift card and 1 ticket together cost $52, then 1 gift card alone would cost $52 – $12 = $40.

Domain: Functions

In grade seven, students learned to determine if two quantities represented a proportional relationship. Proportional reasoning is a transitional topic, coming between arithmetic and algebra. Underlying the progression from proportional reasoning through algebra and beyond is the idea of a *function*— a rule that assigns to each input exactly one output. In grade eight a critical area of instruction is the concept of a function. Students are introduced to functions, and they learn proportional relationships are part of a broader group of linear functions.

Functions	8.F

Define, evaluate, and compare functions.

1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[5]
2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*
3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line;

[5] Function notation is not required in Grade 8.

give examples of functions that are not linear. *For example, the function A = s^2 giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

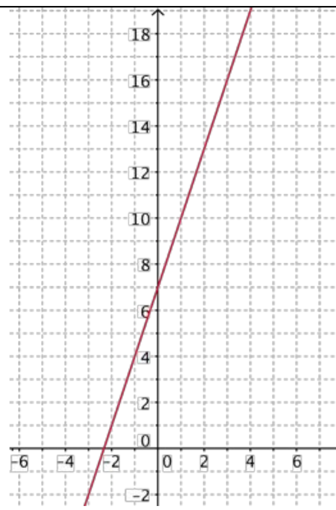
In grade eight, students understand two main points in regards to functions (**8.F.1▲**):

- a *function* is a rule that assigns to each input exactly one output, and
- the *graph* of a function is the set of ordered pairs consisting of an input and the corresponding output.

In general, students understand that functions describe situations in which one quantity determines another. The main work in grade eight concerns linear functions, though students are exposed to non-linear functions to contrast them with linear functions. Thus, students may view a linear equation like $y = -.75x + 12$ as a rule that defines a quantity y whenever the quantity x is given. In this case, the function may describe the amount of money remaining after x turns when a student who starts with $12 plays a game that costs $.75 per turn. Or, students may view the formula for the area of a circle, $A = \pi r^2$ as a (non-linear) function in the sense that the area of a circle is dependent on its radius. Student work with functions at grade eight remains informal, but sets the stage for more formal work in the higher mathematics courses.

Example: Introduction to Functions.
To introduce the concept of a function, a teacher might have students contrast two workers' wages at two different jobs, one with an hourly wage and the other based on a combination of hourly wage and tips. Students read through scenarios and make a table for each of the two workers, listing hours worked and money earned during 20 different shifts varying from 3 to 8 hours in length. Students answer questions about the data, including the level of predictability of the wage of each worker, based on the number of hours worked. Students graph the data and observe the patterns of the graph. Next, the teacher could introduce the concept of a function and relate the tables and graphs from the activity to the idea of a function, emphasizing that an input value completely determines an output value. Students could then be challenged to find other quantities that are functions and to create and discuss corresponding tables and/or graphs.

Students are able to connect foundational understandings about functions to their work with proportional relationships. The same kinds of tables and graphs students used in seventh grade to recognize and represent proportional relationships between quantities are used in eighth grade when students compare the properties of two functions that are represented in different ways (e.g., numerically in tables, visually in graphs). Students also compare the properties of two functions that are represented algebraically or verbally (**8.F.2▲**).

Example: Functions Represented Differently. Which function has a greater rate of change?		
Function 1: The function represented by the graph shown. *Function 2*: The function whose input x and output y are related by the equation $y = 4x + 7$.		**Solution:** The graph of the function shows that when $x = 0$ the value of the function is $y = 7$, while when $x = 2$ the value of the function is $y = 13$. This means that Function 1 increases by 6 units when x increases by 2 units. Function 2 also has an output of $y = 7$ when $x = 0$, but when $x = 2$ the value of Function 2 is $y = 15$. This means that Function 2 increases by 8 units when x increases by 2 units. Therefore, Function 2 has a greater rate of change.

Students' understanding of the equation $y = mx + b$ deepens as they learn that the equation defines a linear function whose graph is a straight line (**8.F.3▲**), a concept closely related to standard (**8.EE.6▲)**. To avoid the mistaken impression that all functional relationships are linear, students also work with nonlinear functions and provide examples of nonlinear functions, recognizing that the graph of a nonlinear function is not a straight line.

Functions	8.F
Use functions to model relationships between quantities.	
4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description or from two (*x, y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a	

table of values.

5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

In grade eight students learn to use functions to represent relationships between quantities. This work is also closely tied to Mathematical Practice (**MP.4 *Model with Mathematics***). There are many situations in real-world problems that can be modeled with linear functions, including instances of constant payment plans (e.g. phone plans), costs associated with running a business, and relationships between certain associated bivariate data (see standard **8.SP.3**). Students also recognize that linear functions in which $b = 0$ are proportional relationships, something they have studied since grade six.

Standard **8.F.4** refers to students finding the *initial value* of a linear function. What is intended by this standard is that if f represents a linear function with a domain $[a, b]$, that is, the input values for f are between the values a and b, then the initial value for f would be $f(a)$. The term "initial value" takes its name from an interpretation of the independent variable as representing time, although the term can apply to any function. Note that formal introduction of the term *domain* does not occur until the higher mathematics courses, but teachers may desire to include this language if it clarifies these ideas for students. The example below will illustrate this definition.

Example: Modeling With a Linear Function. A car rental company charges $45 per day for the car as well as a one-time $25 fee for the car's GPS navigation system. Write an equation for the cost in dollars, c, as a function of the number of days the car is rented, d. What is the initial value for this function?

Solution: There are several aids that may help students determine an equation for the cost:

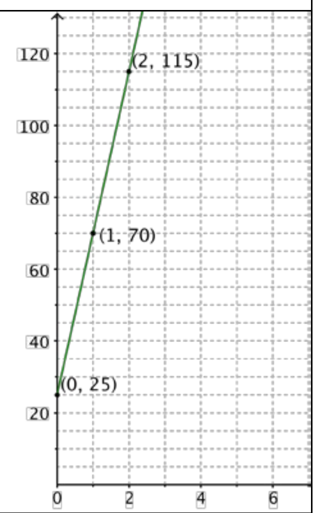
- A verbal description: "Each day adds $45 to the cost, but there is the one-time $25 GPS fee. This means that the cost should be $25 plus $45 times whatever number of days you rent the car. This gives me $c = 25 + 45d$. Since the least number of days you could rent the car is 1 day, the initial value is $25 + 45 = 70$, which means it costs $70 to rent the car for 1 day."

- A table: "I made a table to give me a feel for how much the rental might cost after d days.

d (days)	1	2	3	4	d
c (dollars)	70 $= 25 + (1)45$	115 $= 25 + (2)45$	160 $= 25 + (3)45$	205 $= 25 + (4)45$	25 $+ (d)45$

The table helped me see that the cost in dollars is given by $c = 25 + (d)45$. Since it only makes sense to rent the car for 1 day or more, the initial value is when $d = 1$, which is $c = 70$, or $70."

- A graph: "I made a rough graph and saw that the relationship between the cost and the days rented appeared to be linear. So I found the slope of the line to be 45 (which is the cost per day) and the *y* intercept to be 25. This means the equation is $c = 45d + 25$. What's important to see is that even though the *y* intercept of the graph is 25, that isn't the initial value since that would be when someone rents the car for 1 day. Since the point on the graph is $(1,70)$, the initial value is $70."



Students analyze graphs and then describe qualitatively the functional relationship between two quantities (e.g., where the function is increasing or decreasing, linear or nonlinear). They are able to sketch graphs that illustrate the qualitative features of functions that are described verbally (**8.F.5**).

[Note: Sidebar]

Focus, Coherence, and Rigor:

Work in the cluster "Use functions to model relationships between quantities" involves functions for modeling linear relationships and computing a rate of change or initial value, which supports major work at grade eight with proportional relationships and setting up linear equations. (**8.EE.5-8▲**)

Domain: Geometry

In grade seven, students solved problems involving scale drawings and informal geometric constructions, and they worked with two- and three-dimensional shapes to solve problems involving area, surface area, and volume. In grade, eight students complete their work on volume by solving problems involving cones, cylinders, and spheres. The also analyze two- and three-dimensional space and figures using distance, angle, similarity, and congruence and through understanding and applying the Pythagorean Theorem, which is a critical area of instruction at the grade.

Geometry	8.G

Understand congruence and similarity using physical models, transparencies, or geometry software.

1. Verify experimentally the properties of rotations, reflections, and translations:
    a. Lines are taken to lines, and line segments to line segments of the same length.
    b. Angles are taken to angles of the same measure.
    c. Parallel lines are taken to parallel lines.
2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

A major shift in the traditional curriculum occurs in this domain in grade eight with the introduction of basic transformational geometry. In particular, the notion of *congruence* is defined differently than it has been defined in the past. Whereas previously two shapes were taken to be congruent if they had the "same size and same shape," this imprecise notion is exchanged for a more precise one, that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations. Given that this is the case, students need ample opportunities to explore these three geometric transformations and their properties. The work in the grade eight Geometry domain is designed to provide a seamless transition to the Geometry conceptual category in the higher mathematics courses, which begins with transformational geometry from a more advanced perspective.

With the aid of physical models, transparencies, and geometry software, eighth grade students gain an understanding of transformations and their relationship to congruence of shapes. Through experimentation, students verify the properties of rotations, reflections, and translations, including discovering that these transformations change the position of a geometric figure but not its shape or size **(8.G.1a-c▲)**. Finally, students come to understand that congruent shapes are precisely those that can be “mapped” one onto the other by using rotations, reflections or translations.

Characteristics of Rotations, Reflections, Translations.[6] Students come to understand that the following transformations result in shapes that are *congruent* to one another.
Students understand a *rotation* as the spinning of a figure around a fixed point known as the center of rotation. Rotations are usually performed counterclockwise according to a certain angle of rotation, unless otherwise specified.
Students understand a *reflection* as the flipping of an object over a line, known as the line of reflection.
Students understand a *translation* as the shifting of an object in one direction a fixed distance, so that any point lying on the shape moves the same distance in the same direction.

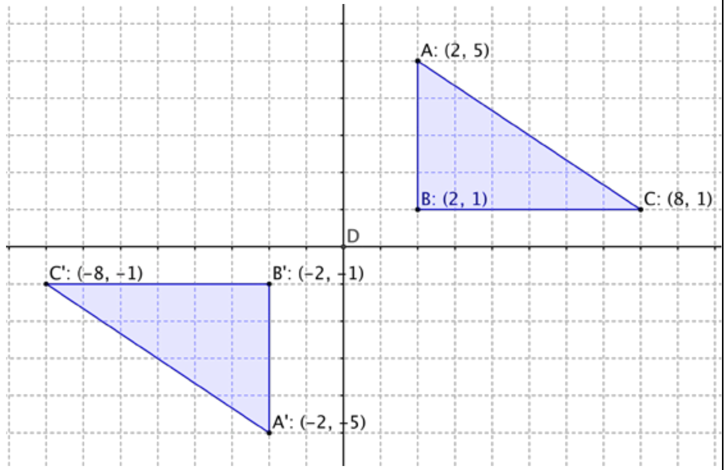
Students also study dilations in standard **(8.G.3▲)**. A *dilation* with scale factor $k > 0$ can be thought of as a stretching (if $k > 1$) or shrinking (if $k < 1$) of an object. In a dilation, a point is specified from which the distance to the points of a figure are multiplied to obtain new points, and hence a new figure.

Examples of Geometric Transformations. (Note that the original figure is called the *preimage*, while the new figure is called the *image*.)

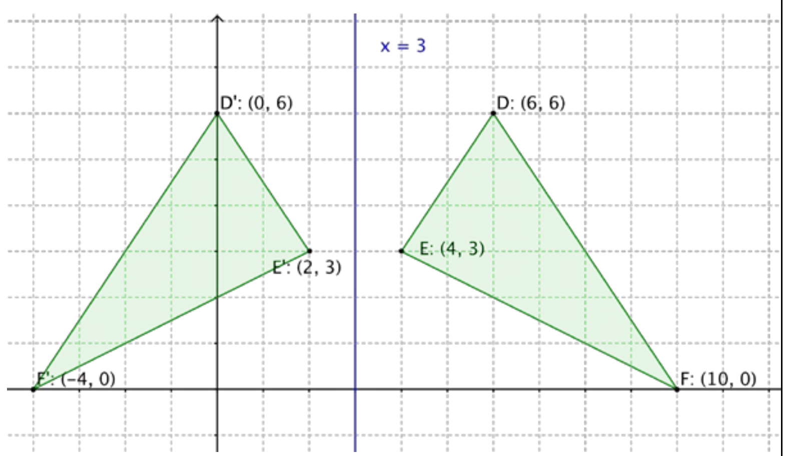
[6] An example of an interactive online tool that shows transformation is “Interactive Transmographer,” which allows students to work with rotation, reflection, and translation. Available at http://www.shodor.org/interactivate/activities/Transmographer/ (Shodor 2013).

Rotation: A figure can be rotated up to 360° about the center of rotation.

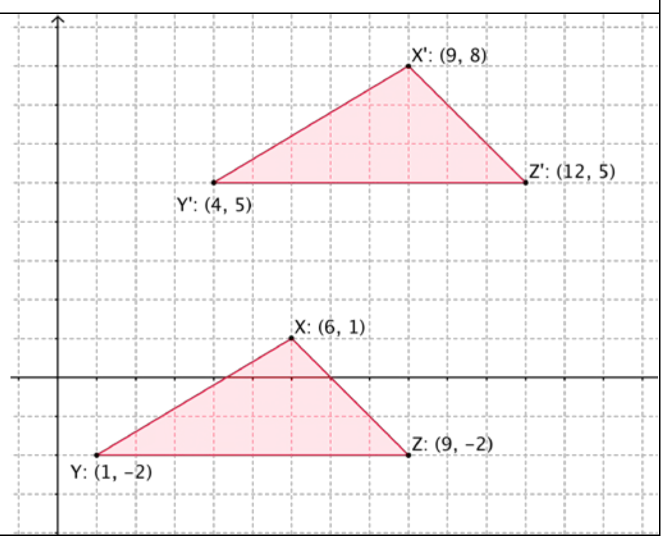
Consider when $\triangle ABC$ is rotated 180° *clockwise* about the origin. The coordinates of $\triangle ABC$ are $A(2,5)$, $B(2,1)$, and $C(8,1)$. When rotated 180°, the image triangle $\triangle A'B'C'$ has coordinates $A'(-2,-5)$, $B'(-2,-1)$, $C'(-8,-1)$. Each coordinate is the opposite of its preimage point's coordinate.



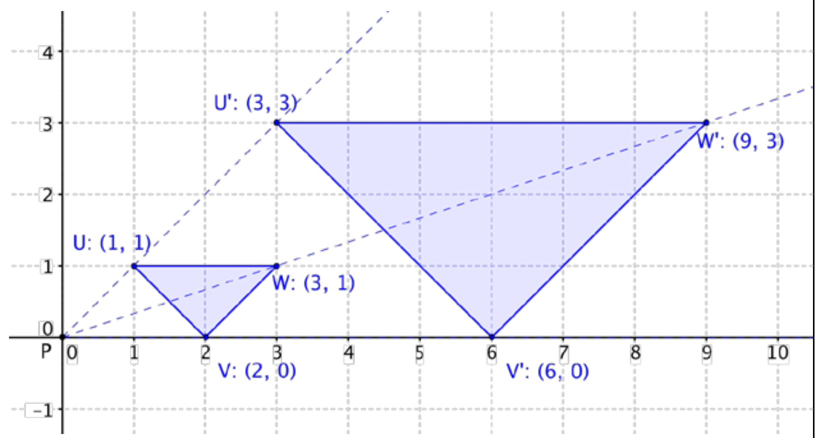
Reflection: In the picture shown, $\triangle DEF$ has been reflected across the line $x = 3$. Notice the change in the orientation of the points, in the sense that the clockwise order of the preimage D-E-F is reversed in the image to D'-F'-E'. Notice also that each point on the image is at a same distance from the line of reflection as its corresponding point on the preimage.



Translation: Here, $\triangle XYZ$ has been translated 3 units to the right and 7 units up. Orientation is preserved. It is not too hard to see that under this transformation, a point (x, y) yields the image point $(x + 3, y + 7)$.



Dilation: In the picture, $\triangle\, UVW$ has been dilated from the origin $P\!: (0,0)$ by a factor of $k = 3$. The picture shows that the segments $OU, OV,$ and OW have all been multiplied by the factor $k = 3$, which results in new triangle, $\triangle\, U'V'W'$. By definition, $\triangle\, UVW$ and $\triangle\, U'V'W'$ are *similar* triangles. Students should experiment and find that the ratios of corresponding side lengths satisfy $\frac{U'V'}{UV} = \frac{V'W'}{VW} = \frac{U'W'}{UW} = 3$, which corresponds to k. Students can apply the



Pythagorean Theorem (**8.G.7-8**) to find the side lengths and justify this result: e.g., by finding the lengths of VW and $V'W'$:

$$\overline{VW} = \sqrt{1^2 + 1^2} = \sqrt{2} \text{ and } \overline{V'W'} = \sqrt{3^2 + 3^2} = \sqrt{18}$$

(Students can check informally that $\sqrt{18} = 3\sqrt{2}$ as formal work with radicals has not yet begun.)

Geometry	8.G

Understand congruence and similarity using physical models, transparencies, or geometry software.

4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

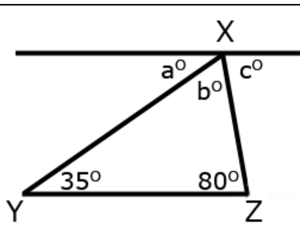
Analogous to the new definition of congruence, the definition of *similar shapes* has been refined to be more precise. Whereas previously shapes were said to be similar if they had the “same shape but not necessarily the same size,” now, two shapes are called *similar* if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations **(8.G.4▲)**. Through investigating dilations and using reasoning such as in the previous example, students find that:

1. When two shapes are similar, the length of a segment AB in the first shape is multiplied by the scale factor k to give the length of the corresponding segment $A'B'$ in the second shape: $\overline{A'B'} = k \cdot \overline{AB}$.
2. Since the previous fact is true for all sides of a dilated shape, the ratio of the lengths any two corresponding sides of the first and second shape is equal to k.
3. It is also true that the ratio of any two side lengths from the first shape is the same as the ratio of the corresponding side lengths from the second shape, e.g., $\frac{\overline{AB}}{\overline{BC}} = \frac{\overline{A'B'}}{\overline{B'C'}}$. (Students can justify this algebraically since fact 2 yields that $\frac{\overline{AB}}{\overline{A'B'}} = \frac{\overline{BC}}{\overline{B'C'}}$.)

Students use informal arguments to establish facts (**8.G.5▲**) about the angle sum and exterior angles of triangles (e.g., consecutive exterior angles are supplementary), the angles created when parallel lines are cut by a transversal (e.g., corresponding angles are congruent), and the angle–angle criterion for similarity of triangles (if two angles of a triangle are congruent to two angles of another triangle, the two triangles are similar). The angle-angle criterion for triangle similarity, when coupled with the previous three properties of similar shapes, allows students to justify the fact that the slope of a line is the same between any two points on the line (see discussion of standard **8.EE.6▲**).

Example: The sum of the measures of the angles of a triangle is 180°.

In the figure shown, the line through point X is parallel to segment YZ. I know that $a = 35$ because it is the measure of an angle that is alternating with angle $\angle Y$. For a similar reason, $c = 80$. Because lines have an angle measure of $180°$, we know that $b = 180 - (35 + 80) = 65°$. So the sum of the measures of the angles in this triangle is $180°$.



Geometry	8.G

Understand and apply the Pythagorean Theorem.

6. Explain a proof of the Pythagorean Theorem and its converse.
7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-

world and mathematical problems in two and three dimensions.

8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

The Pythagorean Theorem is useful in practical problems, relates to grade-level work in irrational numbers, and plays an important role mathematically in coordinate geometry in higher mathematics. In grade eight, students explain a proof of the Pythagorean Theorem (**8.G.6▲**). There are many varied and interesting proofs of the Pythagorean Theorem.[7] In grade eight students apply the theorem to determine unknown side lengths in right triangles (**8.G.7▲**) and to find the distance between two points in a coordinate system (**8.G.8▲**). Work with the Pythagorean Theorem will support students' work in high school-level coordinate geometry.

[Note: Sidebar]

Focus, Coherence, and Rigor:

Understanding, modeling and applying (**MP.4**) the Pythagorean Theorem and its converse requires that student look for and make use of structure (**MP.7**) and express repeated reasoning (**MP.8**). Students also construct and critique arguments as they explain a proof of the theorem and its converse to others (**MP.3**). (Adapted from The Charles A. Dana Center Mathematics Common Core Toolbox 2012).

Geometry	8.G

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

9. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

In grade seven students learned about the area of a circle. In eighth grade students learn the formulas for calculating the volumes of cones, cylinders, and spheres and use the formulas to solve real-world and mathematical problems (**8.G.9**). When students learn to solve problems involving volumes of cones,

[7] For example, geometric "proof without words" of the Pythagorean Theorem available at http://illuminations.nctm.org/activitydetail.aspx?id=30 (NCTM Illuminations 2013).

cylinders and spheres — together with their previous grade seven work in angle measure, area, surface area and volume — they will have acquired a well-developed set of geometric measurement skills. These skills, along with proportional reasoning and multistep numerical problem solving, can be combined and used in flexible ways as part of modeling during high school and in college and careers (Adapted from PARCC 2012).

Domain: Statistics and Probability

Building on work in earlier grades with univariate measurement data and analyzing data on line plots and histograms, eighth-grade students begin to work with bivariate measurement data and use scatter plots to represent and analyze the data.

Bivariate measurement data consist of data that represent two measurements. Scatter plots can show the relationship between the two measured variables. Collecting and analyzing bivariate measurement data help students to answer questions such as how does more time spent on homework affect test grades and what is the relationship between years of education and annual income.

Statistics and Probability	8.SP

Investigate patterns of association in bivariate data.

1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*
4. Understand that patters of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class*

on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?

Students construct and interpret scatter plots to investigate patterns of association between two quantities (**8.SP.1**). Students build on their previous knowledge of scatter plots to examine relationships between variables. They analyze scatter plots to determine positive and negative associations, the degree of association, and type of association. Students examine outliers to determine if data points are valid or represent a recording or measurement error.

Example: Creating Scatter Plots.

Customer satisfaction is vital to the success of fast food restaurants, and speed of service is a key component of that satisfaction. In order to determine the best staffing level, the owners of a local fast food restaurant have collected the data below showing the number of staff members and the average time for filling an order. Describe the association between the number of staff and the average time for filling an order, and make a recommendation as to how many staff should be hired.

Number of staff	3	4	5	6	7	8
Average time to fill order (seconds)	180	138	120	108	96	84

Students can use tools such as those at the National Center for Educational Statistics (NCES) to create a graph or generate data sets, available at http://nces.ed.gov/nceskids/createagraph/default.aspx (NCES Kids Zone 2013).

Students know that straight lines are widely used to model relationships between two quantitative variables (**8.SP.2**). For scatter plots that appear to show a linear association, students informally fit a line (e.g., by drawing a line on the coordinate plane between data points) and informally assess the fit by judging the closeness of the data points to the straight line.

Example: Informally Determining a Line of Best Fit.

The capacity of the fuel tank in a car is 13.5 gallons. The table below shows the number of miles traveled and how many gallons of gas are used. Describe the relationship between the variables. If the data is linear, determine a line of best fit. Do you think the line represents a good fit for the data set? Why or why not? What is the average fuel efficiency of the car in miles per gallon?

Miles Traveled	0	75	120	160	250	300
Gallons Used	0	2.3	4.5	5.7	9.7	10.7

Students solve problems in the context of bivariate measurement data by using the equation of a linear model (**8.SP.3**). They interpret the slope and the y-intercept. For example in a linear model for a biology experiment, students interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in the height of the plant.

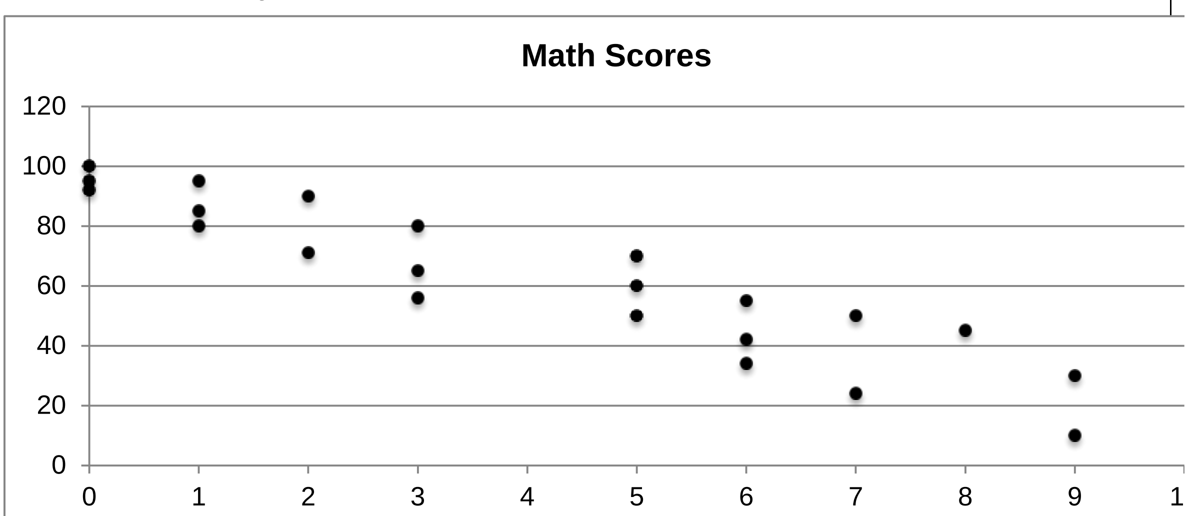
Example: Finding a Linear Model for a Data Set.

Given data from students' math scores and absences, make a scatter plot. Informally fit a line to the graph, and determine an approximate linear function that models the data. What would you expect to be the score of a student with 4 absences?

Solution:

Absences	3	5	1	1	3	6	5	3	0	7	8
Math Scores	65	50	95	85	80	34	70	56	100	24	45
Absences	2	9	0	6	6	2	0	5	7	9	1
Math Scores	71	30	95	55	42	90	92	60	50	10	80

Students would most likely make a scatter plot using simple data software, and find a graph that looks like the following:



Students can use graphing software to find a line of best fit. Such a line might be $y = -8x + 95$. They interpret this equation as defining a function the gives the approximate score of a student based on the number of absences they have. We would expect a student with 4 absences to have a score of approximately $y = -8(4) + 95 = 63$.

(Adapted from CDE Transition Document 2012, Arizona 2012, and N. Carolina 2013)

[Note: Sidebar]

Focus, Coherence, and Rigor:

In eight grade students apply their experience with coordinate geometry and linear functions to plot bivariate data as points on a plane and to make use of the equation of a line in analyzing the relationship between two paired variables. Students develop mathematical practices as they build statistical models to explore the relationship between two variables (**MP.4**) and look for and make use of structure to describe possible association in bivariate data (**MP.7**) (Adapted from Progressions 6-8 SP 2011).

Students learn to see patterns of association in bivariate categorical data in a two-way table (**8.SP.4**). They construct and interpret a two-way table that summarizes data on two categorical variables collected from the same subjects. The two-way table displays frequencies and relative frequencies. Students use relative frequencies calculated from rows or columns to describe a possible association between the two variables. For example, students collect data from their classmates about whether they have a curfew and whether they do chores at home. The two-way table allows students to easily see if students who have a curfew also tend to do chores at home.

Example: Two-Way Tables for Categorical Data.

The table illustrates the results when 100 students were asked the survey questions: (1) Do you have a curfew? (2) Do you have assigned chores?

		Curfew	
		Yes	No
Chores	Yes	40	10

Students can examine the survey results to determine if there is evidence that those who have a curfew also tend to have chores.

No	10	40

Solution: Of the students who answered that they had a curfew, 40 had chores and 10 did not. Of the students who answered they did not have a curfew, 10 had chores and 40 did not. From this sample, there appears to be a positive correlation between having a curfew and having chores since it appears that most students with chores have a curfew and most students without chores do not have a curfew.

[Note: Sidebar]

Focus, Coherence, and Rigor:

Work in the Statistics and Probability cluster “Investigate patterns of association in bivariate data” involves looking for patterns in scatterplots and using linear models to describe data. This is directly connected to major work in the Expressions and Equations clusters (**8.EE.1-8▲**) and also provides opportunities for students to model with mathematics (**MP.4**).

(Adapted from CDE Transition Document 2012, Arizona 2012, and N. Carolina 2013)

A detailed discussion of statistics and probability is provided online at Draft 6–8 Progression on Statistics and Probability (Progressions 6-8 SP 2011).

Essential Learning for the Next Grade

In middle school, multiplication and division develop into powerful forms of ratio and proportional reasoning. The properties of operations take on prominence as arithmetic matures into algebra. The theme of quantitative relationships also becomes explicit in grades six through eight, developing into the formal notion of a function by grade eight. Meanwhile, the foundations for later courses in deductive geometry are laid in the middle grades. The gradual development of data representations in kindergarten through grade five leads to statistics in middle school: the study of shape, center, and spread of data distributions; possible associations between two variables; and the use of sampling in making statistical decisions.

In higher mathematics courses, algebra, functions, geometry, and statistics develop with an emphasis on modeling. Students continue to take a thinking approach to algebra, learning to see and make use of structure in algebraic expressions of growing complexity (Adapted from PARCC 2012).

To be prepared for the higher mathematics courses students should be able to demonstrate they have acquired certain mathematical concepts and procedural skills by the end of grade eight. Prior to grade eight, some standards identify fluency expectations at the grade level. In eighth grade linear algebra is an instructional focus and although the grade eight standards do not specifically identify fluency expectations, eighth grade students who can fluently solve linear equations (**8.EE.7▲**) and pairs of simultaneous linear equations (**8.EE.8▲)** will be better prepared to complete courses in higher mathematics. These fluencies and the conceptual understandings that support them are foundational for work in higher mathematics. Students have been working informally with one-variable linear equations since as early as kindergarten. This important line of development culminates in grade eight with the solution of general one-variable linear equations, including cases with infinitely many solution or no solutions as well as cases requiring algebraic manipulation using properties of operations.

Of particular importance for students to attain in grade eight are skills and understandings to work with radical and integer exponents **(8.EE.1-4▲);** understand connections between proportional relationships, lines, and linear equations **(8.EE.5-6▲);** analyze and solve linear equations and pairs of simultaneous linear equations **(8.EE.7-8▲);** and define, evaluate, and compare functions **(8.F.1-3▲)**. In addition, the skills and understandings to use functions to model relationships between quantities (**8.F.4-5**) will better prepare students to use mathematics to model real-world problems in the higher grades.

Guidance on Course Placement and Sequences

The standards support a progression of learning. Many culminating standards that remain important far beyond the particular grade level appear in the middle grades. As stated in the Common Core standards:

"…some of the highest priority content for college and career readiness comes from grades 6–8. This body of material includes powerfully useful proficiencies such as applying ratio reasoning in real-world and mathematical problems, computing fluently with positive and negative fractions and decimals, and solving real-world and mathematical problems involving angle measure, area, surface area, and volume…"

(CCSSI 2010, Note on courses and transitions).

The Common Core standards for grades 6 – 8 are comprehensive, rigorous, and non-redundant. Acceleration will require compaction not the former strategy of deletion. Therefore, careful consideration needs to be made before placing a student into higher mathematics coursework in middle grades. Acceleration may get students to advanced coursework but might create gaps in students' mathematical background. Careful consideration and systematic collection of multiple measures of individual student performance on both the content and practice standards will be required. For additional information and guidance on course placement, see "Appendix A: Course Placement and Sequences" in this framework.

Grade 8 Overview

The Number System

- Know that there are numbers that are not rational, and approximate them by rational numbers.

Expressions and Equations

- Work with radicals and integer exponents.
- Understand the connection between proportional relationships, lines, and linear equations.
- Analyze and solve linear equations and pairs of simultaneous linear equations.

Functions

- Define, evaluate, and compare functions.
- Use functions to model relationships between quantities.

Geometry

- Understand congruence and similarity using physical models, transparencies, or geometry software.
- Understand and apply the Pythagorean Theorem.
- Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

Statistics and Probability

- Investigate patterns of association in bivariate data.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 8

The Number System 8.NS

Know that there are numbers that are not rational, and approximate them by rational numbers.

1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.
2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π_2). *For example, by truncating the decimal expansion of* $\sqrt{2}$*, show that* $\sqrt{2}$ *is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

Expressions and Equations 8.EE

Work with radicals and integer exponents.

1. Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.
2. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.
3. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as* 3×10^8 *and the population of the world as* 7×10^9*, and determine that the world population is more than 20 times larger.*
4. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Understand the connections between proportional relationships, lines, and linear equations.

5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*
6. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Analyze and solve linear equations and pairs of simultaneous linear equations.

7. Solve linear equations in one variable.
   a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).
   b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
8. Analyze and solve pairs of simultaneous linear equations.
   a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
   b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example,* $3x + 2y = 5$ *and* $3x + 2y = 6$ *have no solution because* $3x + 2y$ *cannot simultaneously be 5 and 6.*
   c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

Functions 8.F

Define, evaluate, and compare functions.

1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[1]
2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*
3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function* $A = s^2$ *giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

Use functions to model relationships between quantities.

4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Geometry 8.G

Understand congruence and similarity using physical models, transparencies, or geometry software.

1. Verify experimentally the properties of rotations, reflections, and translations:
    a. Lines are taken to lines, and line segments to line segments of the same length.
    b. Angles are taken to angles of the same measure.
    c. Parallel lines are taken to parallel lines.
2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

Understand and apply the Pythagorean Theorem.

6. Explain a proof of the Pythagorean Theorem and its converse.
7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

9. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

[1]Function notation is not required in Grade 8.

Statistics and Probability 8.SP

Investigate patterns of association in bivariate data.

1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*
4. Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*